Asymptotic behavior of ultimately contractive iterated random Lipschitz functions

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Abstract

Let $(F_n)_{n\geq 0}$ be a random sequence of i.i.d. global Lipschitz functions on a complete separable metric space $(\mathbb{X}, d)$ with Lipschitz constants $L_1, L_2, \ldots$ For $n \geq 0$, denote by $M_n^x = F_n \circ \ldots \circ F_1(x)$ and $\hat{M}_n^x = F_1 \circ \ldots \circ F_n(x)$ the associated sequences of forward and backward iterations, respectively. If $\mathbb{E}\log^+ L_1 < 0$ (mean contraction) and $\mathbb{E}\log^+ d(F_1(x_0), x_0)$ is finite for some $x_0 \in \mathbb{X}$, then it is known (see [4]) that, for each $x \in \mathbb{X}$, the Markov chain M_n^x converges weakly to its unique stationary distribution π, while $\hat{M}_n^x$ is a.s. convergent to a random variable $\hat{M}_\infty$ which does not depend on x and has distribution π. In [1], a renewal theoretic approach has been successfully employed to provide convergence rate results for $\hat{M}_n^x$ which then also lead to corresponding assertions for M_n^x via $M_n^x \stackrel{d}{=} \hat{M}_n^x$ for all n and x, where $\stackrel{d}{=}$ means equality in law. A refinement of this approach leads to similar results in the more general situation where only ultimate contraction, i.e. an a.s. negative Lyapunov exponent $\lim_{n\to\infty} n^{-1} \log l(F_n \circ \ldots \circ F_1)$ is assumed (here $l(F)$ denotes the Lipschitz constant of F). Our talk intends to give a survey of the results as well as the main techniques.

References

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