

Regular variation, the weak law and the St. Petersburg game

Allan Gut, Uppsala University

Abstract

The celebrated Kolmogorov-Feller *weak* law of large numbers holds if and only if $nP(|X| > n) \rightarrow 0$ as $n \rightarrow \infty$, where X is a generic summand. An interesting point is that this condition is weaker than the assumption of a finite mean, which, in turn, is necessary and sufficient for the *strong* law to hold.

In the first part of the talk we present an extension of the K-F weak law to distributions for which the tail of the i.i.d. summands is regularly varying with index $\alpha \in (0, 1]$. The proof turns into a simplified version of the so-called classical degenerate convergence criterion. As an application we show that the weak law for the St. Petersburg game is an immediate consequence of this result.

In the second part we consider some recent generalizations of the game.

References

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Allan Gut, Department of Mathematics, Uppsala University, Box 480, SE-751 06 Uppsala, Sweden;
Email: allan.gut@math.uu.se URL: <http://www.math.uu.se/~allan>