Renewal theorems for some weighted renewal functions

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Joint results of Indlekofer, Klesov and myself is presented.

Let X, X_1, X_2, \ldots be a family of integer valued, independent and identically distributed random variables, $0 < \mu = EX$, $\sigma^2 = E(X - \mu)^2$. Let $S_n = X_1 + \ldots + X_n$, $R(k) = \sum_{n=1}^{\infty} a_n P(S_n = k)$, $a_n \ge 0$, $a_n = O(n^{\varepsilon})$. Assume that for $A(x) = \sum_{n=1}^{\infty} a_n$ the condition $c_1 hL(x) \le A(x+h) - A(x) \le c_2 hL(x)$, $c_2 \le \frac{L(h)}{2} \le c_4$

 $=\sum_{\substack{n \le x} a_n} \text{ the condition } c_1 hL(x) \le A(x+h) - A(x) \le c_2 hL(x), \ c_3 \le \frac{L(h)}{L(x)} \le c_4$

hold with $L(x) \ge 1$ $(x \ge 1)$ and $\sqrt{x} \le h \le x$.

The asymptotic of R(k) is given under some special, arithmetically characterized choice of a_n .