Regular variation of linear operators: An overview with limit theorems in mind

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Abstract: Regular variation techniques have been proved extremely useful for limit theorems in probability theory of various kind. Often the norming sequences in certain multivariate limit theorems can be chosen as regularly varying linear operators $A_n \in \mathrm{GL}(\mathbb{R}^d)$ of some index $E \in \mathrm{GL}(\mathbb{R}^d)$, i.e.

$$\lim_{n\to\infty} A_{\lfloor \lambda n \rfloor} A_n^{-1} = \lambda^E \quad \text{for all } \lambda > 0,$$

where $\lambda^E = \exp(E \log \lambda)$ is the usual matrix exponential.

Although some of the basic properties of regular variation in the univariate case $d = 1$ do no longer hold in the multivariate situation, it is remarkable that most of the key results in the one-variable theory can be extended to the operator case. The talk will give an overview on these key results with special emphasis on questions motivated from limit theorems. In particular we will show how regular variation techniques of linear operators can be successfully applied to capture possible symmetries of a limit law and to prove sharp estimates of the growth behavior into any radial direction.