Multivariable Regular Variation of measures: An overview

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Abstract: The theory of multivariate regular variation of measures is a powerful tool to analyze limit theorems of matrix-normed sums of i.i.d. random vectors on $\mathbb{R}^d$. The theory is based on regular variation of linear operators, which is a matrix-valued extension of the classical one-dimensional theory of regular variation.

We say that a Borel measurable function $f : \mathbb{R}^+ \to \mathrm{GL}(\mathbb{R}^d)$ varies regularly with index $-E \in \mathrm{GL}(\mathbb{R}^d)$ if

$$\lim_{t\to\infty} f(\lambda t)f(t)^{-1} = \lambda^{-E} \tag{0.1}$$

for all $\lambda > 0$, where $\lambda^E = \exp(E \log \lambda)$ is the matrix exponential.

Moreoever, a finite Borel measure μ on $\mathbb{R}^d$ is called regularly varying with index $E \in \mathrm{GL}(\mathbb{R}^d)$ if there exists a regularly varying function f as in (0.1) such that

$$t \cdot \big(f(t)\mu\big) \to \phi \quad \text{vaguely for } t \to \infty, \tag{0.2}$$

for a certain σ-finite measure ϕ on $\mathbb{R}^d \setminus \{0\}$. Here $\big(f(t)\mu\big)(A) = \mu\big(f(t)^{-1}A\big)$ denotes the image measure. The limit measure ϕ in (0.2) then satisfies

$$\lambda \cdot \phi = \big(\lambda^E \phi\big) \quad \text{for all } \lambda > 0.$$

In the talk we give an overview of some key results of the above mentioned theories.