Assembling for FE Methods on a Planar Domain

By means of an example[a] we explain how the assembling algorithm in a finite-element procedure sets up the working matrix A. Figure 1 presents a planar domain tiled by $m = 12$ triangles $\mathcal{D}_k$, $k = 1, \ldots, m$. A numeration of the m triangles and of N nodes[b] (vertices) is given, and $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ are colored in red, blue, and green. These colors will repeat in Figures 2 and 3, and will help illustrating the procedure. The numbering is chosen such that the bandwidth of the resulting global $(N \times N)$-matrix $A = (a_{ij})$ will become small. Relations between the N nodes and m triangles are collected in an integer matrix $\mathcal{I}$ of three columns and m rows. The row $\mathcal{I}_k$ assigns the numbers of the three vertices of $\mathcal{D}_k$. Its first entries are

$$\begin{array}{llll} \mathcal{I}_1: & 2 & 1 & 3 \\ \mathcal{I}_2: & 4 & 2 & 6 \\ \mathcal{I}_3: & 2 & 3 & 6 \\ \mathcal{I}_4: & 6 & 3 & 5 \end{array}$$

Assembling procedure: Start from a zero matrix A; run a loop $k = 1, \ldots, m$. Each triangle $\mathcal{D}_k$ carries local information in a (3×3)-matrix[c], whose elements are distributed additively to nine specific positions of the global $(N \times N)$-matrix A. Target positions (i, j) of the assembling routine are defined in $\mathcal{I}_k$, see Figure 2.

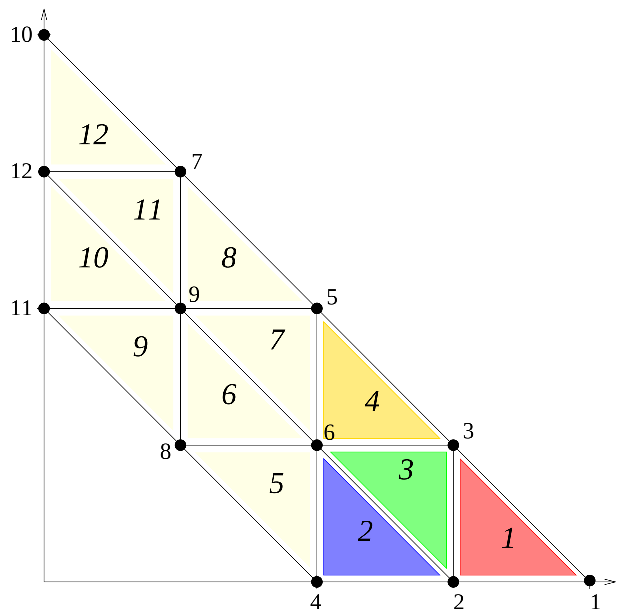

Figure 1: The domain tiled into triangles

[a] Example 5.5, Exercise 5.8 (domain for a basket with double barrier)

[b] In this triangulation, m and N happen to be equal, $m = N = 12$.

[c] E.g., its elements are the nine integrals of the element-stiffness matrix.

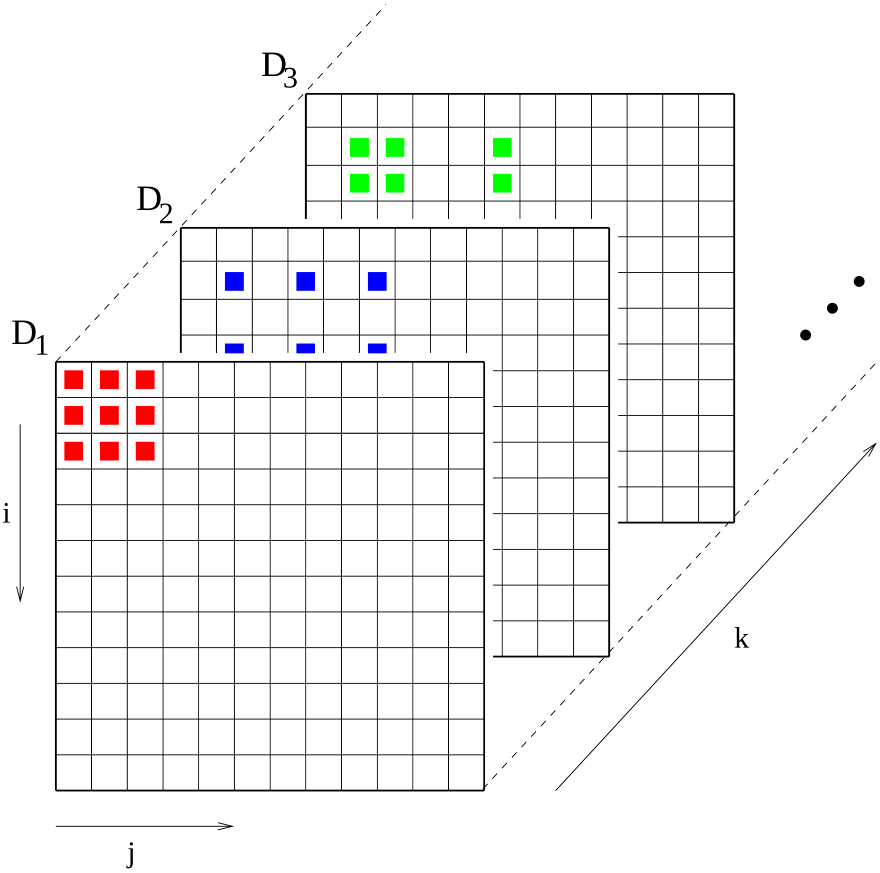

Figure 2: Local information is assembling.

In Figure 3 the assembled elements of the first four triangles are indicated; several elements of the global matrix are covered frequently. To show all entering colors we use different forms of symbols. The black squares wait for more assembling input (the reader may complete). Remaining elements are zero, nothing is assembled there. The bandwidth becomes visible: $a_{ij} = 0$ for $|i - j| > 5$.

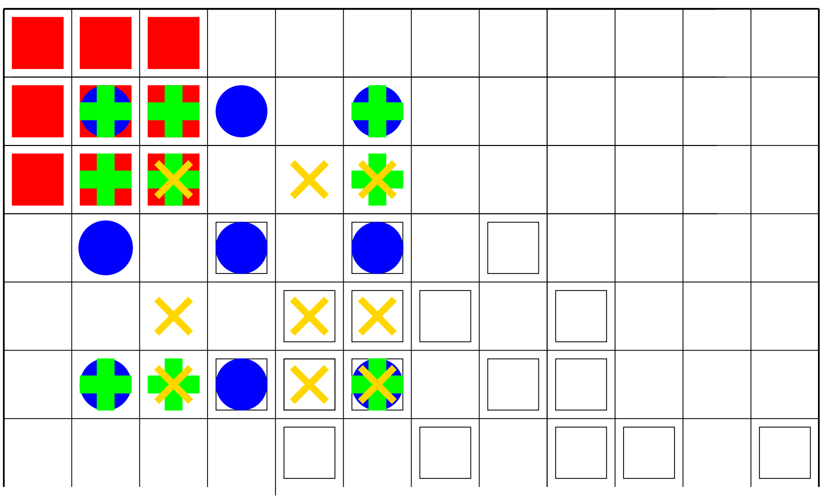

Figure 3: Upper part of the global matrix, colors as above