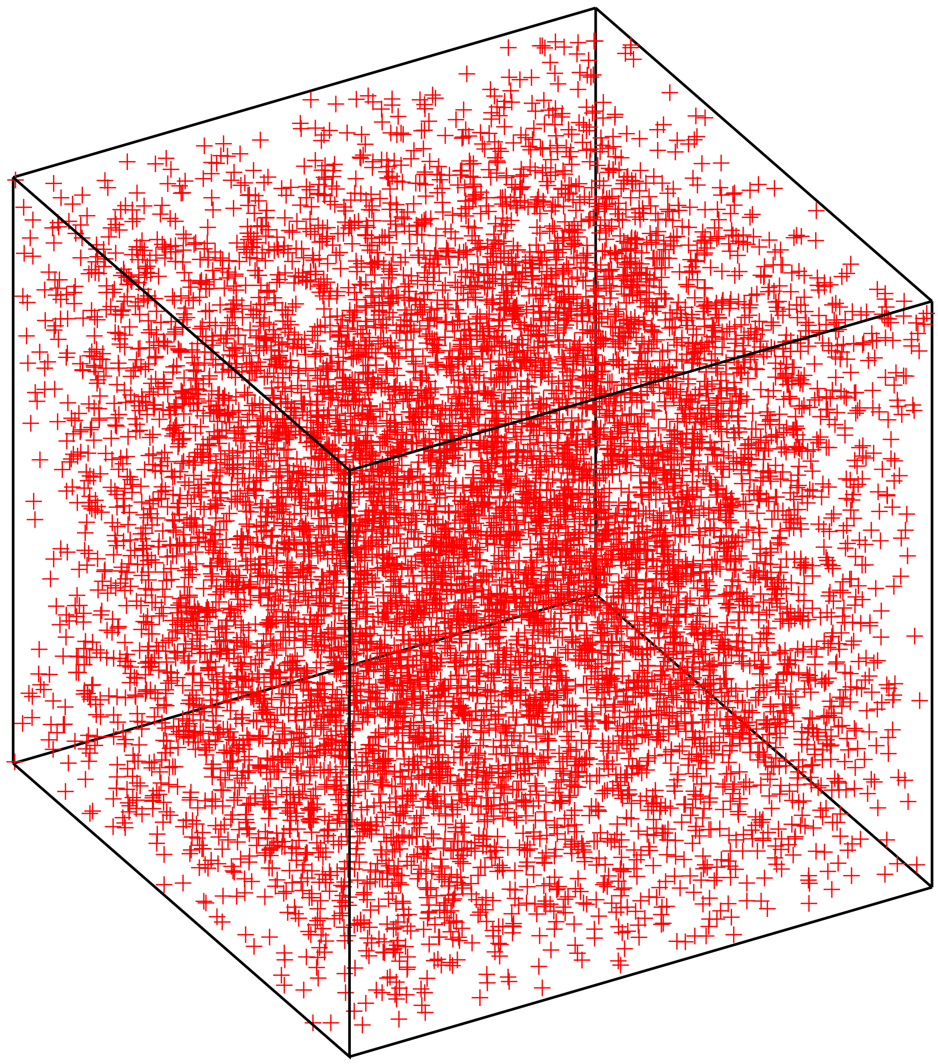
Random Numbers from RANDU

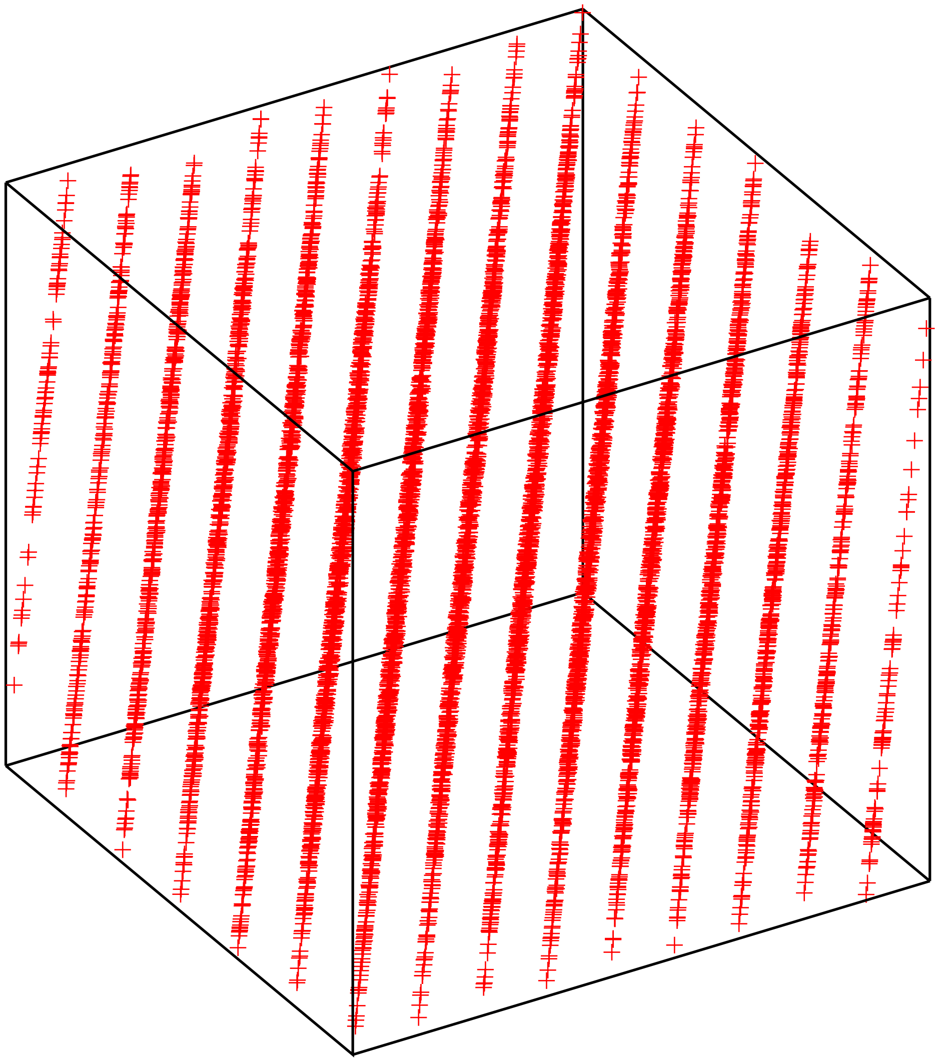
The recursion "RANDU"

$$N_i = aN_{i-1} \bmod M, \quad \text{with } a = 2^{16} + 3, \; M = 2^{31}$$

for integers N_i and $i = 1, 2, \ldots$ defines a sequence of real numbers $U_i := N_i/M$. These may serve as pseudo-random numbers. Perfect random numbers subsequently arranged as m-tuples or points in $\mathbb{R}^m$ would fill the unit cube in a rather equidistributed way. For $m = 2$ experiments with RANDU show that the points (U_{i-1}, U_i) appear nicely equidistributed in the square. For $m = 3$ the figure below depicts a view with 998 points (U_{i-2}, U_{i-1}, U_i) in the cube $[0, 1]^3$. This view on the experiment might suggest good results of the RANDU generator.



But rotating the cube $[0,1]^3$ by 90 degrees reveals that pseudo-random points (U_{i-2}, U_{i-1}, U_i) in the cube fall on only 15 planes. This must be seen as a weakness of the RANDU generator. Imagine as application, for example, a Monte-Carlo integration over a solid in the cube. The points would put unequal weight on different parts of the solid.



The views in the above figures are defined in `gnuplot` by 60,60 for the first plot and 60,149 for the second plot.