Rejection Method: Applications

Assume two densities f and g such that for a constant c the inequality $f \leq cg$ holds. Assume further that a random number generator for g is available, and one for f is required. The **rejection-acceptance algorithm** is:

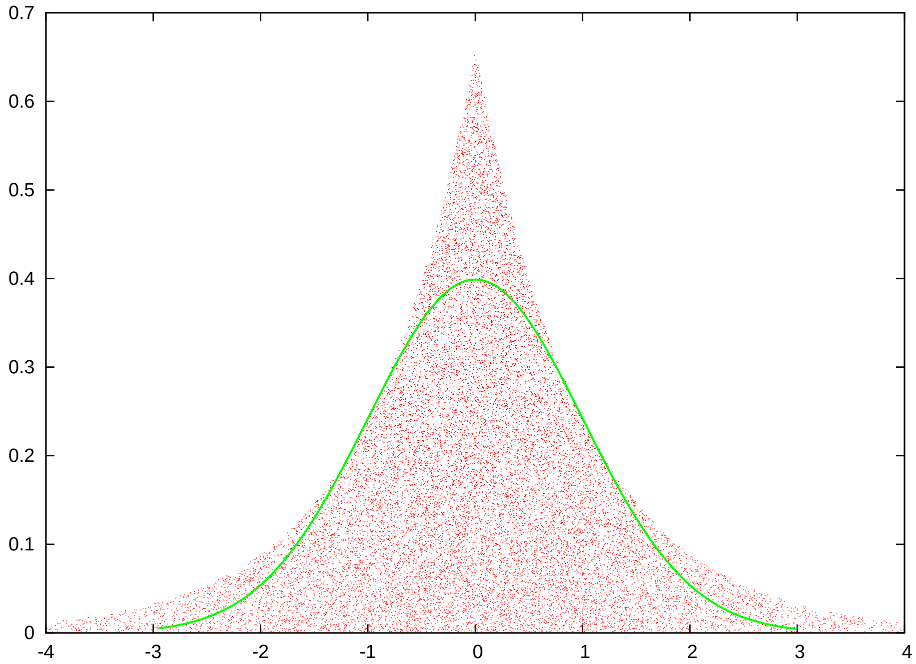
repeat
  $x :=$ random number distributed according g,
  $u :=$ random number $\sim \mathcal{U}[0,1]$
until $u \cdot c \cdot g(x) < f(x)$
return x

Example: Laplace Distribution

The figure shows almost 20000 points with Laplace-distributed x-coordinate, with density

$$g(x) := \frac{1}{2}\exp(-|x|)$$

(a few points outside the interval $-4 \leq x \leq 4$ are cut off). The y-coordinates are $u \cdot g(x)$, with $u \sim \mathcal{U}[0,1]$. For the density f of the standard normal distribution $c = 1.3155$ works. (check it!)



The normal density function f is plotted in green. All points under that curve are accepted and have x-coordinates distributed normally. The points above the curve are rejected.

Example: Ziggurat 8

The figure shows in green the normal density function $f(x)$ for $0 \leq x \leq 4$. Further it illustrates a "ziggurat" with $N = 8$ horizontal layers. The bounding curve g of the ziggurat is shown in red. All g-distributed points inside the area indicated in blue need not be fully calculated, only their x-coordinates matter. For larger N this leads to a highly efficient algorithm, the *ziggurat*.