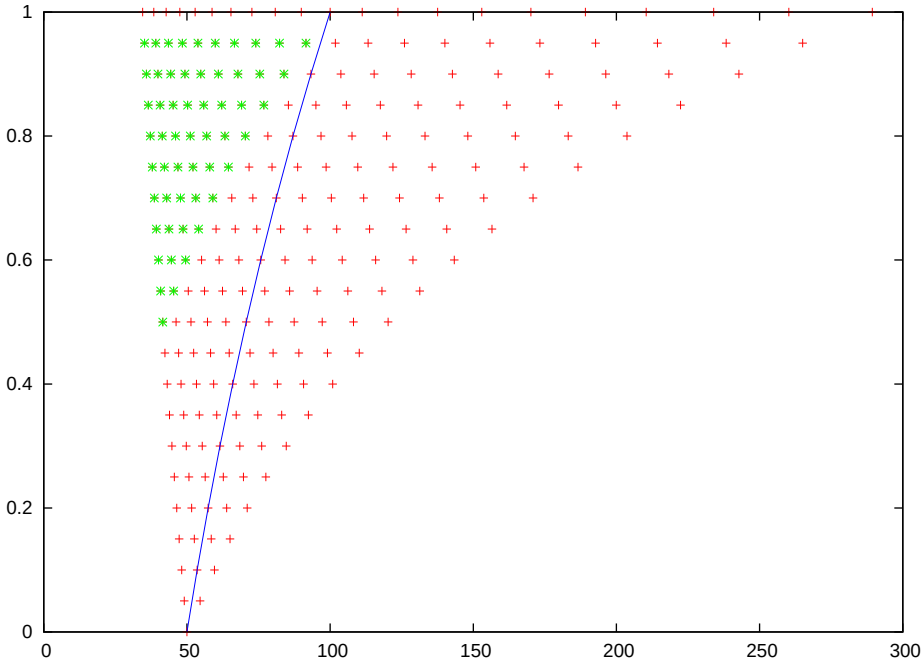
Tilted Tree for Vanilla Options

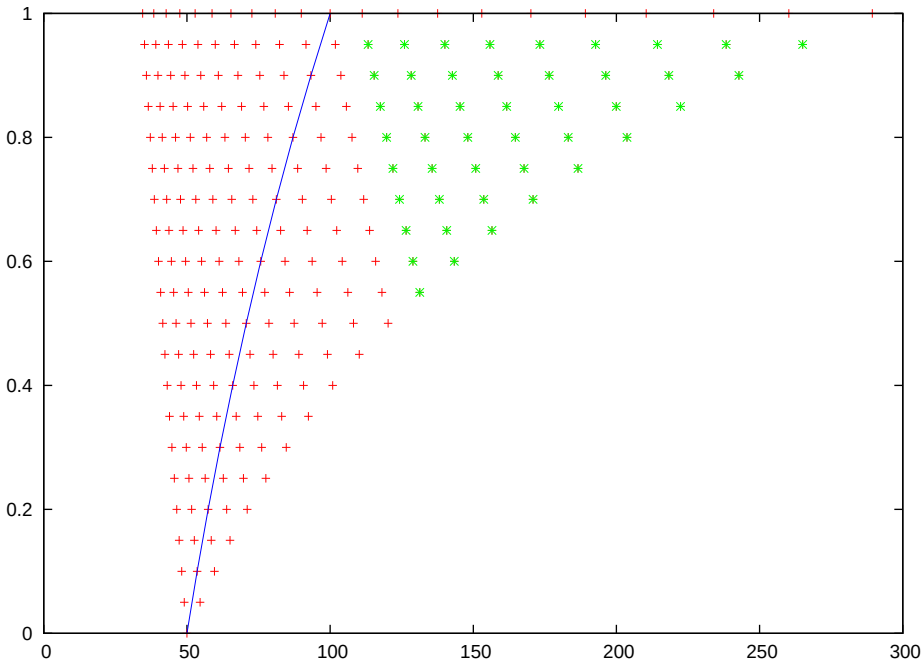
The standard tree, but modified so that the center node at maturity T hits the strike K, is shown in the two figures below. The example uses an underlying price $S_0 = 50$, strike $K = 100$, maturity $T = 1$, and $M = 20$ time levels. The tree is determined with $ud = \exp(\frac{2}{M} \log \frac{K}{S_0})$, its core line is shown in blue. Further, those nodes for which the pricing results in $V = 0$ are indicated by green stars. That is, one quarter of the nodes are not needed, and the effort can be reduced accordingly.

The figures show the the tree in the (S, t)-plane.

call

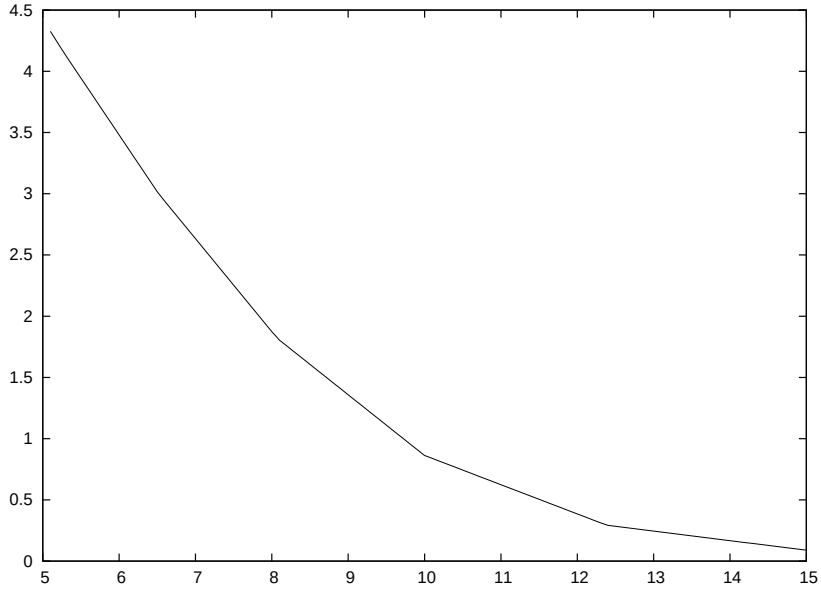


put

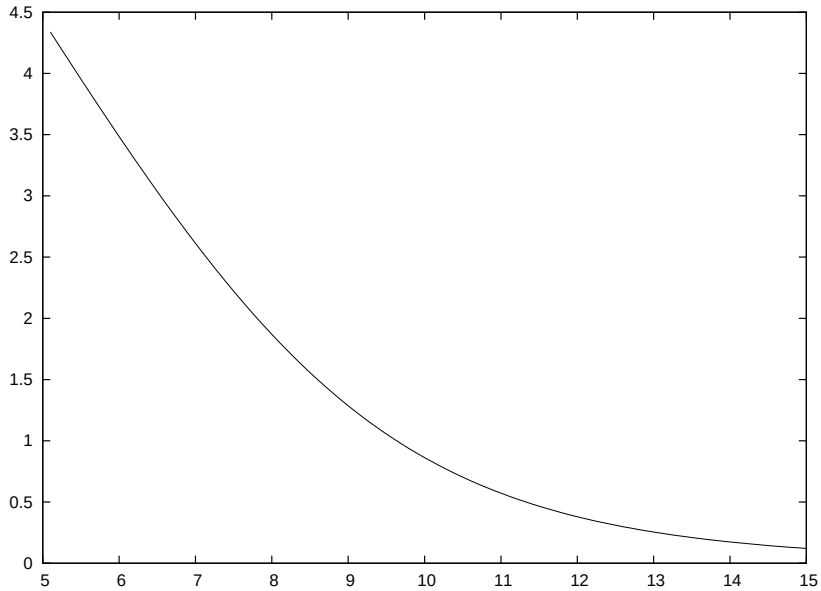


Smoothing the Results of the Binomial Tree

Valuation along the tree yields the value $V^{(M)}$ for given S. The tilting has the effect of smoothing with respect to S. This is illustrated by the two figures below, which depict the function $V^{(M)}(S)$ of a vanilla put with strike $K = 10$ for $t_0 = 0$ as depending on the initial price $S = S_0$, here for $M = 8$, and $5 \leq S \leq 15$.



$V^{(M)}(S)$ is piecewise linear for the classical binomial tree $[ud = 1]$.



$V^{(M)}(S)$ is smooth for the tilted tree $[ud = \exp(\frac{2}{M} \log \frac{K}{S})]$.

The tilting was suggested by Leisen & Reimer (1996); see also Exercise 1.14 in [Tools fCF].