MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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Summer 2011
April 6

Computational Finance - 1st Assignment

Deadline: April 13

Exercise 1 (Portfolios) (5+5 points)

In a financial market the following securities are traded: a share S as well as put and call options (all related to S) with three strike prices $K_1 \leq K_2 \leq K_3$.

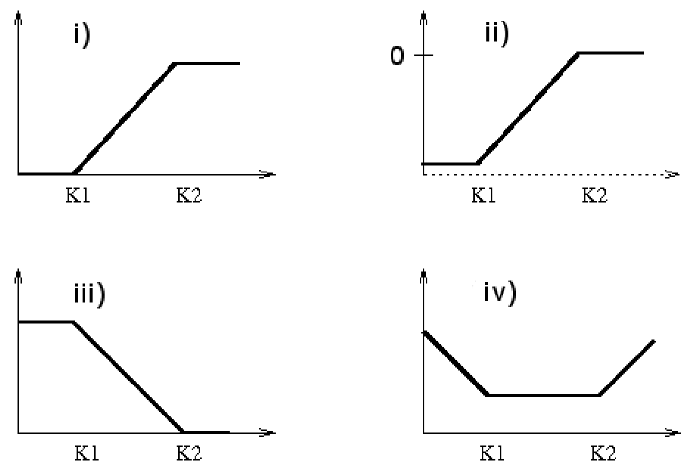
a) Sketch the payoffs of the following portfolios. What is the maximum profit or loss in each case?

   i) Put with strike K_1 as a short and as a long position respectively.

   ii) Call with strike K_1 as a short and as a long position respectively.

   iii) One call with strike K_1 (long), two calls with strike K_2 (short) and one call with strike K_3 (long).

b) For each of the following payoffs, construct portfolios out of vanilla options such that the payoff is met. (In example ii) note that the S-axis is shifted.)



Exercise 2 (No-Arbitrage Principle, Put-Call Parity) (3+7 points)

a) Use arbitrage arguments to prove the following bounds on option prices:

   i) $V_{\mathrm{C}} \geq 0$

   ii) $S \geq V_{\mathrm{C}}^{\mathrm{am}} \geq (S-K)^+$

b) Consider a portfolio consisting of three positions related to the same asset, namely one share (price S), one European put (value $V_{\mathrm{P}}^{\mathrm{eur}}$), plus a short position of one European call (value $V_{\mathrm{C}}^{\mathrm{eur}}$). Put and call have the same expiration date T, and no dividends are paid.

i) Show that the *put-call parity*

$$S + V_{\mathrm{P}}^{\mathrm{eur}} - V_{\mathrm{C}}^{\mathrm{eur}} = Ke^{-r(T-t)}$$

holds for all t, where K is the strike and r the risk-free interest rate.

ii) Use the put-call parity to show

$$V_{\mathrm{C}}^{\mathrm{eur}}(S,t) \geq S - Ke^{-r(T-t)},$$
$$V_{\mathrm{P}}^{\mathrm{eur}}(S,t) \geq Ke^{-r(T-t)} - S.$$

Information:

- The first exercise course takes place on Monday, 18th of April.
- Additional information can be found at

www.mi.uni-koeln.de/~seydel/numerik/SoSe2011/NuFi.html .