MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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Computational Finance - 12th Assignment

Deadline: July 6

Exercise 37 (Stability of the Fully Implicit Method) (4 points)

The backward-difference method is defined via the solution of the equation

$$A_{\text{impl}}\, w^{(\nu)} = w^{(\nu-1)} \quad \text{with} \quad A_{\text{impl}} = \text{tridiag}(-\lambda,\ 2\lambda+1,\ -\lambda).$$

Prove the stability.

Hint: Use $w^{(\nu)} = A_{\text{impl}}^{-1}\, w^{(\nu-1)}$.

Exercise 38 (Front-Fixing for American Options) (4+2 points)

Apply the transformation

$$\zeta := \frac{S}{S_{\text{f}}(t)}\,, \quad y(\zeta, t) := V(S, t)$$

to the Black-Scholes equation.

a) Show
$$\frac{\partial y}{\partial t} + \frac{\sigma^2}{2}\zeta^2 \frac{\partial^2 y}{\partial \zeta^2} + \left[(r-\delta) - \frac{1}{S_{\text{f}}}\frac{dS_{\text{f}}}{dt}\right]\zeta\frac{\partial y}{\partial \zeta} - ry = 0.$$

b) Set up the domain for (ζ, t) and formulate the boundary conditions for an American call. (Assume $\delta > 0$.)

[Project: Set up a finite-difference scheme to solve the derived boundary-value problem. The curve $S_{\text{f}}(t)$ is implicitly defined by the above PDE, with final value $S_{\text{f}}(T) = \max(K, \frac{r}{\delta}K)$.]

Exercise 39 (Upwind Scheme) (5 points)

Apply von Neumann‘s stability analysis to

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = b\frac{\partial^2 u}{\partial x^2}\,, \quad a > 0, \quad b > 0$$

using the FTBS upwind scheme for the left-hand side and the centered second-order difference quotient for the right-hand side.