

Computational Finance - 2nd Assignment

Deadline: April 20

Exercise 3 (Lipschitz Continuity) (4 points)

Show that the payoff of a vanilla call option defined by $(S - K)^+$ is Lipschitz continuous.

Exercise 4 (Black-Scholes Formula) (5+15P points)

For a European call the analytic solution of the Black-Scholes equation is given by

$$d_1 := \frac{\log \frac{S}{K} + \left(r - \delta + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}},$$
$$d_2 := d_1 - \sigma\sqrt{T - t} = \frac{\log \frac{S}{K} + \left(r - \delta - \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}},$$
$$V_C(S, t) = Se^{-\delta(T-t)}F(d_1) - Ke^{-r(T-t)}F(d_2),$$

where F denotes the standard normal cumulative distribution (compare a)), and δ is a continuous dividend yield. The value $V_P(S, t)$ of a European put is obtained by applying the put-call parity

$$V_P = V_C - Se^{-\delta(T-t)} + Ke^{-r(T-t)},$$

from which

$$V_P = -Se^{-\delta(T-t)}F(-d_1) + Ke^{-r(T-t)}F(-d_2)$$

follows.

- a) Establish an algorithm to calculate

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{t^2}{2}\right) dt.$$

Hint: Construct an algorithm to calculate the *error function*

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$$

and use $\operatorname{erf}(x)$ to calculate $F(x)$. Use a trapezoidal sum as a quadrature method.

- b) Write a computer program in Java, C++, C or Fortran that calculates prices of European call and put options using the above Black-Scholes formula and the algorithm constructed in a). Then compute the following option prices at time $t = 0$:
- i) European put with $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$, $S = 5$, $\delta = 0$.
 - ii) European call with the same parameters.

Exercise 5 (European Options with Infinite Maturity)

(3+2 points)

Calculate the following limits:

$$\text{a) } \lim_{T \rightarrow \infty} V_P^{\text{eur}}(S, t), \quad \text{b) } \lim_{T \rightarrow \infty} V_C^{\text{eur}}(S, t).$$

Hint: Consider options with finite maturity and use the bounds of Exercise 2.

Information:

- You are allowed to work on the programming exercises with a partner.
- The deadline for the programming exercise is April 20. Please turn in a printed version of your code and send it to *numerik_programm@gmx.de*.