MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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# Computational Finance - 4th Assignment

**Deadline: May 4**

**Exercise 10 (Anchoring the Binomial Grid at K)** (3+3 points)

The equation $ud = 1$ has established a kind of symmetry for the grid. As an alternative, one may anchor the grid in another way by choosing (for even $M$)

$$S_0 u^{M/2} d^{M/2} = K.$$

a) Give a geometrical interpretation.

b) Derive the relevant formula for $u$ and $d$.
*Hint:* Use Exercise 8 a).

**Exercise 11 (Properties of a Wiener Process)** (5 points)

In the definition of a Wiener process the requirement (b) $W_0 = 0$ is dispensable. Then the requirement in (c) reads

$$W_t - W_0 \sim \mathcal{N}(0, t).$$

Use this relation to deduce for $t > s$:

$$\begin{aligned} &\mathrm{E}(W_t - W_s) = 0, \\ &\mathrm{Var}(W_t - W_s) = \mathrm{E}((W_t - W_s)^2) = t - s. \end{aligned}$$

**Exercise 12 (Binomial Model)** (20P points)

Write a computer program in C++, C, Java or Fortran that calculates prices of options using the binomial model. Create a subroutine which consists of the algorithm of the lecture. The subroutine should have the following input values:

$r$ (interest rate), $\sigma$ (volatility), $T$ (maturity), $K$ (strike), $S_0$ (value of the asset at $t = 0$), $M$ (number of time steps), *type*1 (European/American option), *type*2 (put/call).

The today‘s value $V_0$ of the option should be the output value.

Calculate the following option values for $M = 8, 16, 32, 64$:

a) European put with $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$, $S = 5$.

b) European call with the same parameters.

c) American put with $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$, $S = 9$.

Calculate the difference between $V_C^{\text{eur}}$ and $V_0^{(M)}$, where $V_C^{\text{eur}}$ is the value obtained via the Black-Scholes formula, for example b) and consecutive $M = 50, \ldots, 100$. Use $V_C^{\text{eur}} \approx 0.01282$.

**Information:**

- The deadline for the programming exercise is May 11. Please turn in a printed version of your code and send it to *numerik_programm@gmx.de*.