

Computational Finance - 7th Assignment

Deadline: May 25

Exercise 20 (Greeks)

(4 points)

Consider the Black Scholes model. Determine $\Delta := \frac{\partial V}{\partial S}$ and $\Gamma := \frac{\partial^2 V}{\partial S^2}$ for an European call and put option. Proceed as follows: calculate both greeks for an European call option and use the put call parity to get Δ , Γ for an European put option.

Exercise 21 (Analysis of a Random Number Generator)

(5 points)

Consider the linear congruential generator

$$N_i = (455N_{i-1} + 23) \bmod 4096, \quad U_i = \frac{N_i}{4096}.$$

Construct a family of parallel straight lines containing all the points (U_{i-1}, U_i) so that only few of them cut the square $[0, 1)^2$. What is the distance between them?

Hint: Examine the condition $c \in \mathbb{Z}$ for the linear equation. For this purpose consider the quotient $\frac{M}{a}$.

Exercise 22 (Inverting the Normal Distribution)

(3+1+2 points)

Suppose $F(x)$ is the standard normal distribution function. Construct a rough approximation $G(u)$ to $F^{-1}(u)$ for $0.5 \leq u < 1$ as follows:

- Construct a rational function $G(u)$ with correct asymptotic behavior, point symmetry with respect to $(u, x) = (0.5, 0)$, using only one parameter.
- Fix the parameter by interpolating a given point $(x_1, F(x_1))$.
- What is a simple criterion for the error of the approximation?

Exercise 23 (Uniform Distribution)

(6 points)

For the uniformly distributed random variable (V_1, V_2) on $V_1^2 + V_2^2 < 1$ consider the transformation

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} V_1^2 + V_2^2 \\ \frac{1}{2\pi} \arg(V_1, V_2) \end{pmatrix}.$$

Show that (X_1, X_2) is uniformly distributed.