MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
Prof. Dr. Rüdiger Seydel
Dipl.-Math. Christian Jonen

CCFinance
Cologne Computational Finance Laboratory

Winter 2009/10
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Computational Finance 2 - 1st Assignment

Deadline: October 19

Exercise 1 (Trinomial Tree Method)

In contrast to a binomial tree, from each node of a trinomial tree, we can move to one of three adjacent nodes one period later, see figure 1.

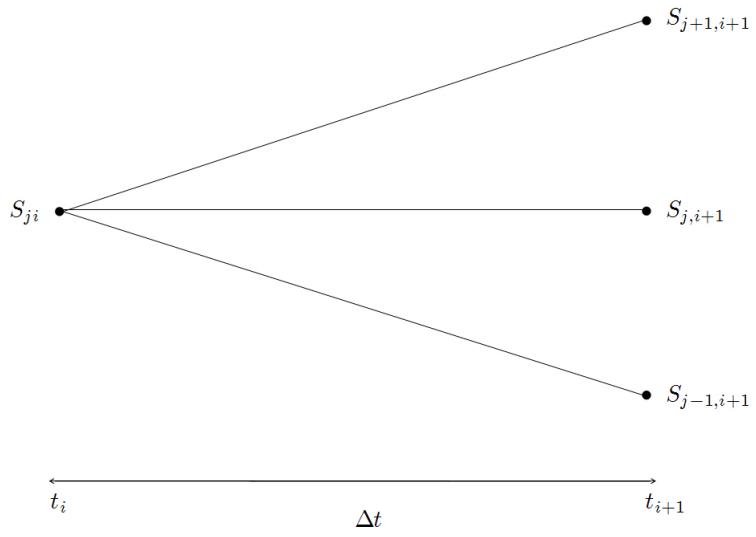

Figure 1: Structure of a trinomial tree.

Let us assume that the tree is recombining, that is, we let

$$S_{j,i+1} = S_{ji},$$
$$S_{j+1,i+1} = uS_{ji},$$
$$S_{j-1,i+1} = dS_{ji},$$
$$ud = 1.$$

Furthermore, we assume that we are in the Black-Scholes world.

a) To derive a system of equations for the unknown parameters (p_u, p_m, p_d, u, d), the jump sizes and probabilities must match the first two moments of the distribution. Remember that we have

$$\mathbb{E}(S_{i+1}) = S_i e^{r\Delta t}$$

for the first moment and

$$Var(S_{i+1}) = S_i^2 e^{2r\Delta t}(e^{\sigma^2 \triangle t} - 1)$$

for the second moment.

b) There is an unlimited number of sample spaces and corresponding probabilities which can be used to price options consistently. A common choice is $p_m = 2/3$. Then, the other parameters can be calculated by using the equations derived in a). In so doing, terms of higher order than Δt may be ignored. However, let us consider the CRR binomial tree method with time step size $\Delta t/2$. By combining two steps of the CRR binomial tree, we obtain one step of a trinomial tree. What is the value of an European option at an arbitrary node S and time $i\Delta t$?

c) Consider the log-transformed Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial x^2} + \left(r - \frac{\sigma^2}{2}\right)\frac{\partial V}{\partial x} - rV = 0.$$

Show that the trinomial tree method is equivalent to the explicit finite difference scheme in the sense of neglecting higher order terms than Δt. By doing so, you get the following differential equation

$$V_j^i = \frac{1}{1 + r\Delta t}(aV_{j+1}^{i+1} + bV_j^{i+1} + cV_{j-1}^{i+1}).$$

Determine the coefficients a, b, c. To interpret a, b, c as probabilities, these three coefficients should all be non-negative and the sum of them must be 1. To this end, find suitable conditions on $\triangle x$ and $\triangle t$.

Information:

Times: Monday from 10:00 a.m. to 11:30 a.m. in S2

Working requirements and assessment method: Exercises are held weekly beginning from the second lecture week. The first exercise course takes place on Monday, 19th of October. At least 50% of the points in the assignments must be achieved to be admitted to the written exam at the end of the course.

Homepage: http://www.mi.uni-koeln.de/∼seydel/NuFi09/NuFi.html

Contact: Christian Jonen, E-Mail: cjonen@math.uni-koeln.de