## **CCFinance** Cologne Computational Finance Laboratory

Winter 2009/10 January 14

## Computational Finance 2 - 10th Assignment

## Deadline: January 21

Exercise 22 (Initial Value Problem)

Consider the ODE initial-value problem

$$u' = -u, \quad u(0) = 2$$

with the additional condition

$$u(t) \ge 1.$$

a) Give an analytical solution.

b) Discuss for a value of  $\epsilon$  with  $0 < \epsilon \ll 1$  the initial-value problem

$$v' = -v + \frac{\epsilon}{v - 1 + \epsilon}, \quad v(0) = 2.$$

Hint: Do some numerical experiments.

c) Show that the solution of the initial-value problem in b) satisfies

$$1 \le v(t) \le 2, \quad v' \le 0, \quad v'' \ge 0,$$

for  $t \geq 0$ .

## Exercise 23 (Fourier Transform)

Consider the Fourier transform

$$\mathcal{F}_f(u) := \int_{-\infty}^{\infty} e^{iuy} f(y) dy.$$

For the example  $f(y) := e^{-a|y|}$  discuss the convergence of

$$\int_{-A}^{A} e^{iuy} f(y) dy$$

for  $A \to \infty$ .

(3+3+4 Points)

(6 Points)