MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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Winter 2009/10
October 29

Computational Finance 2 - 3rd Assignment

Deadline: November 5

Exercise 5 (Galerkin FE Approach for European Options) (4 Points)

By applying the Galerkin FE approach to the transformed version $y_\tau = y_{xx}$ of the Black-Scholes equation, we yield the following system of ordinary differential equations

$$B\dot{w} + b = -Aw - a, \tag{1}$$

where the vectors a, b and the matrices A, B are defined as in the lecture. Apply the Crank-Nicolson discretization scheme to (1).

Exercise 6 (Construction of a Mesh) (3+4+1 Points)

Consider the domain

$$\mathcal{D} := \{(x, y) | 1 \leq x + y \leq 2\}$$

tiled into 12 triangles Ω_k, $k = 1, \ldots, 12$, where triangles and nodes are numbered as in figure 1.

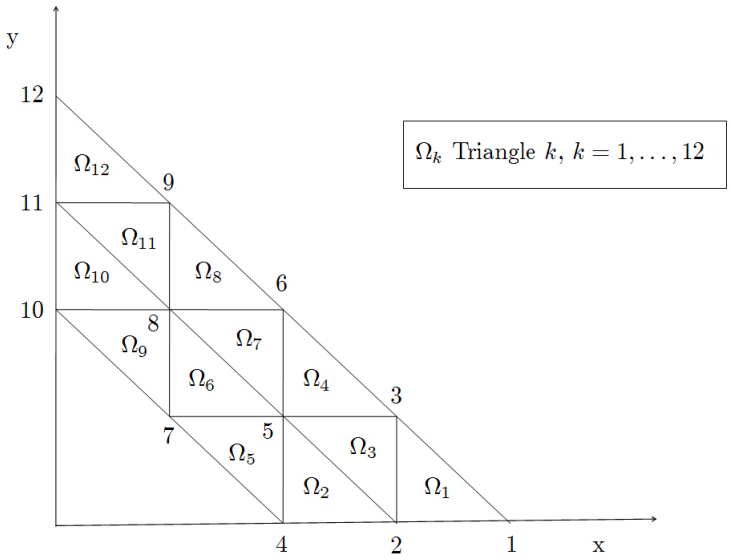

Figure 1: Mesh for the domain $\mathcal{D} := \{(x, y) | 1 \leq x + y \leq 2\}$ tiled into 12 triangles Ω_k, $k = 1, \ldots, 12$.

a) Set up the index set Π with entries

$$\Pi_k = \{i_{k,1}, i_{k,2}, i_{k,3}\},$$

which assigns node numbers to the k-th triangle Ω_k for $1 \leq k \leq 12$.

b) Formulate the assembling algorithm that builds up the stiffness matrix out of the element stiffness matrices

$$\begin{pmatrix} s_{11}^{(k)} & s_{12}^{(k)} & s_{13}^{(k)} \\ s_{21}^{(k)} & s_{22}^{(k)} & s_{23}^{(k)} \\ s_{31}^{(k)} & s_{32}^{(k)} & s_{33}^{(k)} \end{pmatrix}$$

for a general index set and $1 \leq k \leq m$.

c) The example of figure 1 leads to a banded stiffness matrix. What is its bandwidth?

Exercise 7 (Barrier for Options on Two Assets) (3+5 Points)

Suppose the situation of two assets S_1 and S_2. Let their movement for $t > 0$ be governed by Brownian motion with initial price point $(S_1(0), S_2(0))$. Barriers can be aligned such that the probability that $(S_1(t), S_2(t))$ reaches the barrier has the same constant value.

a) Show that this curve of constant probability has an elliptical shape.

b) Let the covariance matrix be

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

Calculate its eigenvalues λ_1, λ_2. Sketch representative ellipses. How do they depend on ρ?