MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
Prof. Dr. Rüdiger Seydel
Dipl.-Math. Christian Jonen

CCFinance
Cologne Computational Finance Laboratory

Winter 2009/10
November 5

Computational Finance 2 - 4th Assignment

Deadline: November 12

Exercise 8 (Error of an Interpolating Polygon) (2+2+2 Points)

For $u \in \mathcal{C}^2$ let u_I be an arbitrary interpolating polygon and h the maximal distance between two consecutive nodes. Show

a) $\max\limits_x |u(x) - u_I(x)| \leq \frac{h^2}{8} \max\limits_x |u''(x)|$,

b) $\max\limits_x |u'(x) - u_I'(x)| \leq h \max\limits_x |u''(x)|$,

c) the assertion $\|u - w_h\|_1 = \mathcal{O}(h)$, where h and w_h denote the maximum mesh size and the discrete solution, respectively.

Exercise 9 (Front Fixing Finite Element Approach) (5+5 Points)

Remember that the price $V(S,t)$ of an American call option is the solution of the following free boundary value problem:

$$V_t + 0.5\sigma^2 S^2 V_{SS} + (r-\delta)SV_S - rV = 0,\ 0 < S < S_f(t),\ 0 \leq t < T,$$

$$V(0,t) = 0,\ 0 \leq t \leq T,$$
$$V(S_f(t),t) = S_f(t) - K,\ 0 \leq t \leq T,$$
$$V_S(S_f(t),t) = 1,\ 0 \leq t \leq T,$$
$$V(S,T) = (S-K)^+,\ 0 \leq S \leq S_f(t).$$

a) Consider the variable transforms

$$V(S,t) = Ke^{-\alpha y - \beta\tau}\phi(y,\tau),\ \tau = T - t,\ S = Ke^y,$$

where α and β are constants, and apply this change of variables to the free boundary value problem above. Sketch the domain of interest.

b) The problem resulting from a) has to be solved on a bounded domain. To this end, by taking numerical aspects into consideration, an artificial boundary may be constructed. According to a), a further straightforward step leads to the problem

$$\psi_\tau - \gamma\psi_{yy} + \nu\psi_y + \mu\psi = 0,\ a(\tau) < y < b(\tau),\ 0 < \tau \leq T,$$

$$\psi(a(\tau),\tau) = 0,\ 0 \leq \tau \leq T,$$

$$\psi(b(\tau),\tau) = g(b(\tau),\tau),\ \ 0 \le \tau \le T,$$
$$\psi_y(b(\tau),\tau) = g_y(b(\tau),\tau),\ \ 0 \le \tau \le T,$$
$$\psi(y,0) = g(y,0),\ \ a(\tau) < y < b(\tau),$$

where $\gamma = 0.5\sigma^2$, $\nu = \gamma(1+2\alpha)+\delta-r$, $\mu = r+\alpha(r-\delta)-\gamma\alpha(1+\alpha)-\beta$, $g(y,\tau) = e^{\alpha y+\beta\tau}(e^y-1)^+$, $b(\tau) = log(S_f(T-\tau)/K)$, $a(\tau) = b(\tau)-L$, and L denotes a constant. Apply the variable transforms $x = y-a(\tau)$, $u(x,\tau) = \psi(a(\tau)+x,\tau)$. Sketch the domain of interest.