MATHEMATISCHES INSTITUT
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Computational Finance 2 - 7th Assignment

Deadline: December 3

Exercise 16 (Estimating Volatility) (6 Points)

Estimates of the local volatility can be obtained from the implied grid. To this end, the return R is investigated at each node (j, i). For a binomial tree, we have two samples for $R_{j,i}$. Taking the return of the underlying process S in the sense $R = \log(S_{\text{new}}/S_{\text{old}})$, the expectation and variance are

$$\mathrm{E}(R_{j,i}) = p_{j,i} \log \frac{S_{j+1,i+1}}{S_{j,i}} + (1 - p_{j,i}) \log \frac{S_{j,i+1}}{S_{j,i}}$$

and

$$\mathrm{Var}(R_{j,i}) = p_{j,i} \left[\log \frac{S_{j+1,i+1}}{S_{j,i}} - \mathrm{E}(R_{j,i}) \right]^2 + (1 - p_{j,i}) \left[\log \frac{S_{j,i+1}}{S_{j,i}} - \mathrm{E}(R_{j,i}) \right]^2 ,$$

respectively. For the GBM model, the scaling is $\mathrm{Var}(R_{j,i}) = \sigma_{j,i}^2 \Delta t$, which defines the local volatility $\sigma_{j,i}$ at node (j, i).
Show that

$$\sigma_{j,i} = \sqrt{\frac{p_{j,i}(1 - p_{j,i})}{\Delta t}} \log \frac{S_{j+1,i+1}}{S_{j,i}}$$

Exercise 17 (EWMA) (6 Points)

Define

$$\sigma_k^2 := \frac{1}{n} \sum_{i=1}^{n} u_{k-i}^2$$

The data are weighted as follows:

$$\text{Replace} \quad \frac{1}{n} \sum_{i=1}^{n} u_{k-i}^2 \quad \text{by} \quad \sum_{i=1}^{n} \alpha_i u_{k-i}^2 \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i = 1 \, .$$

The EWMA (exponentially weighted moving average, also EMA) method sets

$$\alpha_{i+1} = \lambda \alpha_i \quad \text{with} \;\; 0 < \lambda \leq 1 \, .$$

Show that

$$\sigma_k^2 = \lambda \sigma_{k-1}^2 + (1 - \lambda) u_{k-1}^2 + O(\lambda^n)$$