MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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Computational Finance 2 - 9th Assignment

Deadline: December 17

Exercise 20 (Nonlinear Black–Scholes Models I) (4 Points)

According to the lecture, consider the following nonlinear PDE

$$V_t + 0.5\sigma^2(t, S, V_{SS})S^2V_{SS} + (r - \delta)SV_S - rV = 0,$$

where $\sigma^2(t, S, V_{SS})$ depends on the particular model; r is the risk-free interest rate and δ is the continuous dividend yield. Apply the transforms

$$x = \log(S/K),\ \tau = 0.5\sigma_0^2(T - t),\ u(x, \tau) = e^{-x}V(S, t)/K,$$

with $K > 0$ and a model-dependent parameter σ_0.

Exercise 21 (Nonlinear Black–Scholes Models II) (8 Points)

Consider the fully implicit equation discretizing nonlinear Black–Scholes models. Recall the two criteria for $\tilde{\sigma}^2(x, \tau, s)$ with $s := u_x + u_{xx}$

(i) $\tilde{\sigma}^2(x, \tau, s)s$ is continuous and monotone increasing in s,

(ii) there exists a constant $c_+ > 0$ such that for all s and $\epsilon > 0$

$$\tilde{\sigma}^2(x, \tau, s + \epsilon)(s + \epsilon) \geq \tilde{\sigma}^2(x, \tau, s)s + c_+\epsilon$$

out of the three criteria for monotony.
For

(a) Leland's model of transaction costs, with parameter γ, and

(b) the model of uncertain volatility with $\sigma_{\min} \leq \sigma \leq \sigma_{\max}$,

show that (i) and (ii) are satisfied. What are the constants c_+? For (b), σ^- suffices.