

MODULAR AND QUASIMODULAR FORMS, PARTITIONS, AND REPRESENTATIONS OF SYMMETRIC GROUPS

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Abstract:

In 1995, Robbert Dijkgraaf found a “mirror symmetry in dimension one” statement saying that the generating function counting genus g ramified coverings of a torus was what is now called a quasimodular form on the full modular group (i.e., a polynomial in the Eisenstein series E_2 , E_4 and E_6) of weight $2g - 2$. A mathematical proof of this was given by Kaneko and myself in a small paper in which we also gave the definition and basic properties of quasimodular forms. A few years later, Bloch and Okounkov gave a vast generalization showing that the generating series associated to a wide class of functions on partitions are also quasimodular forms. In the course I will explain this theorem and give a very short proof of it, as well as reviewing the necessary background material on modular and quasimodular forms and on the representation theory of finite groups and in particular of symmetric groups. If time permits, I will also describe some extensions and applications of the theory coming from my recent work with Martin Möller, which was in turn motivated by questions coming from the theory of flat surfaces and their moduli spaces.