Modular-invariance in rational conformal field theory: past, present and future

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This is a *tentative* outline of my two lectures. *No* prior knowledge of vertex rings is assumed, though the foundations will be covered only sketchily due to time constraints. The main purpose of the lectures is to explain how modularity enters into VOA theory, and to discuss an approach to the general problem of modular-invariance in rational CFT based on a theory of vector-valued modular forms, Fuchsian systems, and the Riemann-Hilbert problem.

Optional background reading on VOAs:
U. Heidelberg, downloadable lecture notes on connections between VOAs and the Monster, (http://www.cft2011.mathi.uni-heidelberg.de/)
U. Vanderbilt, downloadable lecture notes on VOAs,
(https://my.vanderbilt.edu/ncgoa14)

Outline of talks
1. Vertex rings.
Commutative rings as vertex rings. The center $C(V)$.
2. Locality and quantum fields.
Basic examples: Virasoro algebra, Heisenberg algebra, affine algebra, $V^\natural$.
3. Representations.
Module category, Zhu algebra, Rational CFT, C_2-cofiniteness.
4. Zhu's modular-invariance theorem.
Examples: lattice theories, Virasoro discrete series,
trace functions for $V^\natural$, Jacobi trace functions.
5. Vector-valued modular forms.
Fuchsian equations and the Anderson-Moore theorem.
6. Modular tensor categories.
7. A general modular-invariance conjecture.
8. The algebra of vector-valued modular forms.
Free module theorem, Riemann-Hilbert problem.
9. Modular-invariance conjecture in low dimensions.
Hypergeometric series.
Atkin-Swinnerton-Dyer conjecture on unbounded denominators.