EXERCISES FOR 06.07

Exercise 1

Let Γ be a smooth closed curve contained in the unit sphere $S^2 \subset \mathbb{R}^3$. Let A be the cone over Γ, thus the 2-rectifiable set of all points $\{t \cdot x | 0 \leq t \leq 1, x \in \Gamma\}$. Compute the area $\mathcal{H}^2(A)$ of A using either the are or the co-area formula. What is the 2-density of A at the origin? When is A a C^1-manifold?

Exercise 2

Let V be an m-dimensional affine subspace of $\mathbb{R}^n$. Let $(M, \partial M)$ be a compact m-dimensional submanifold of $\mathbb{R}^n$ contained in V. Prove that M is the minimal filling of ∂M, thus for all $T \in I_{m+1}(\mathbb{R}^n)$ with $\partial T = [\partial M]$ we have $\mathcal{M}(T) \geq \mathcal{M}([M]) = \mathcal{H}^m(M)$. (Hint: Use projection onto V and then "canonical" m-form on V as a calibration form).

Give an example showing that there exist a current $T \in I_n(\mathbb{R}^n)$ which is not minimal, thus there exists some $T' \in I_n(\mathbb{R}^n)$ with $\partial T' = \partial T$ and $\mathcal{M}(T') < \mathcal{M}(T)$.

Exercise 3

Recall the construction of strange Cantor set on Exercise sheet number 5. In the step i we have defined a subset C_i of $\mathcal{R}^2$ consisting of a few equilateral triangles. Consider the 2-current $T_i = [C_i] \in \mathcal{I}(\mathbb{R}^2)$, where we give C_i the canonical orientation of $\mathbb{R}^2$. What is the mass $\mathcal{M}(\partial T_i)$? Explain why ∂T_i is rectifiable. Prove that ∂T_i converges weakly to the 0-current but there is no current T such that ∂T_i converges to T in the $\mathcal{M}$-norm [FIXED].