EXERCISES FOR 22.06

Exercise 1

Recall that the co-mass norm $||\omega||$ of an m-linear form $\omega \in \Lambda^m(\mathbb{R}^n)$ is given as the supremum over all values $\omega(v_1, ..., v_m)$ where all v_i have norm at most 1. Compute the co-mass norms and find corresponding maximizing sequence $(v_1, ..., v_m)$ for

a) $dx_1 + dx_2 \in \Lambda^1(\mathbb{R}^2)$;
b) $dx_1 \wedge dx_2 + dx_2 \wedge dx_3 \in \Lambda^2(\mathbb{R}^3)$;
c) $dx_1 \wedge dx_2 + dx_3 \wedge dx_4 \in \Lambda^2(\mathbb{R}^4)$.

Exercise 2

For the following two currents $T_1, T_2 \in \mathcal{D}_1(\mathbb{R}^2)$ describe the boundaries $\partial T_1, \partial T_2$. Prove that only for one (for which?) of the two cases $i = 1, 2$ there is a number $C > 0$ such that $|\partial T_i(h)| \leq C \cdot \sup_{x \in \mathbb{R}^2} |h(x)|$.

a) T_1 is given as $T_1(\omega) := \int_Q \omega(x)(e_1)\, d\mathcal{H}^2(x)$, where $Q = [0, 1]^2$.
b) T_2 is given as $T_2(\omega) := \int_0^1 \omega(s, 0)\,(e_1 + e_2)\, ds$.