EXERCISES 5

Problem 1. *Consider the following* 2*-dimensional analogue of Cantor set* $A = \cap C_i$ *given by the picture* (C_0 *is a regular triangle and we obtain* C_{i+1} *from* C_i *by removing bunch of regular hexagons). Show that*

(1) $\mathcal{H}^1(P_i(A)) = 1$, *where* P_i *are orthogonal projections to sides of the original triangle* C_0,

(2) $\mathcal{H}^1(A) = 1$.

(3) A *is not* 1*-rectifiable.*

Hint: Use area formula to see that $\forall E \subset \mathbb{R}, \forall f : E \to A$ *Lipschitz map such that* $0 < \mathcal{H}^1(f(E)) < \infty$ *we have* $\mathcal{H}^1((P_1 \circ f)(E)) < \mathcal{H}^1(f(E))$ *or* $\mathcal{H}^1((P_2 \circ f)(E)) < \mathcal{H}^1(f(E))$.

Problem 2. *Construct* $A \subset \mathbb{R}^2$ *such that* A *is* 1*-rectifiable,* $\mathcal{H}^1(A) < \infty$ *and the set of points* $x \in \mathbb{R}^2$ *such that* $\Theta_1(A, x) = 1$ *is dense in* $\mathbb{R}^2$. *Conclude that* $spt(\mathcal{H}^1 \lfloor A) = \mathbb{R}^2$.