applications of orthogonal polynomials and the usefulness of the methods described, this will certainly be the case.

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A Guide to Monte Carlo Simulations in Statistical Physics. Second Edition. *By David P. Landau and Kurt Binder.* Cambridge University Press, Cambridge, UK, 2005. $70.00. xv+432 pp., hardcover. ISBN 0-521-84238-7.

The first edition of this book came out in 2000 and since then has already been cited nearly as often as the now-classic earlier collections of reviews on Monte Carlo methods, edited by K. Binder, were cited in that same time period. This monograph gives a coherent overview of the field, including 20 pages of Fortran 77 programs to learn the details.

The new edition is larger by nearly 50 pages. I counted 18 new sections or subsections in the already existing 12 chapters. The main addition, in my opinion, is the entirely new Chapter 13 on "Monte Carlo Methods Outside Physics." This chapter summarizes protein folding and then, more briefly, other biologically inspired physics, mathematics/statistics, sociophysics, econophysics, traffic simulations, and medicine. (Networks of Watts–Strogatz or Barabasi–Albert type are not mentioned here.) More detailed reviews are cited in nearly all of these sections, and the authors also state that some of these simulations have only tenuous relations to reality. Expert readers may see some work more positively and some more negatively than the two authors, but certainly this new chapter gives a good introduction to this previously "exotic" branch of physics.

Thus I wish this book a third edition (in which German variable names in the bond fluctuation algorithm should be translated) and hope that then the authors will refrain from adding too many new pages to this very useful book.

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Practical Fourier Analysis for Multigrid Methods. *By Roman Wienands and Wolfgang Joppich.* Chapman and Hall/CRC, Boca Raton, FL, 2005. $71.96. xiv+217 pp., hardcover. ISBN 1-58488-492-4.

Myth: Multigrid is inefficient for three-dimensional problems.
[*Several mouse clicks*] 3D Poisson problem, standard second-order finite-difference discretization, $V(1,1)$ cycle with Red-Black relaxation .... And the spectral radius of a three-grid cycle is ... 0.23. Not so bad after all. We expect the complete multigrid V cycle to perform nearly as well, that is, to asymptotically reduce the error by nearly a factor of 0.23 per cycle.

Myth: Successive over-relaxation and multigrid don't mix so well.
[*Click click*] Same problem and methods, but with over-relaxation parameter $\omega = 1.15$, and the spectral radius drops to 0.095, a rather substantial improvement.

Myth: Multigrid is robust for the anisotropic diffusion problem if alternating line relaxation is used.
[*Mouse at work*] Anisotropic diffusion, rotated at 45° relative to the grid, alternating line relaxation in a $V(1,1)$ cycle .... And the spectral radius for the three-grid cycle is ... 0.94. Disappointing! The two-grid spectral radius is 0.75, and the one-grid spectral radius (smoothing factor) is an enticing 0.08. This ominous deterioration with respect to the number of grids is a sure sign that the complete multigrid cycle is going to perform very poorly, due to inadequate coarse-grid correction. A $W(1,1)$ cycle should do somewhat better ... [*click*] ... 0.89. Better, but still unacceptable. Wait, I read somewhere that overweighting the residuals can help ... [*click click*] ... with a residual weighting factor of 1.75, the three-grid spectral radius drops to 0.70. This is worthy of further investigation. But first, let's try a Galerkin coarse-grid discretization, W(1,1) cycle, no residual over-weighting ... [*click*] ... 0.72 for the three-grid cycle. Ok, one more try: didn't someone claim that higher order transfers should be employed in (Petrov) Galerkin discretization of singular perturbation problems? Clicking over to bi-

cubic interpolation we obtain . . . 0.33 for the three-grid spectral radius.

The above describes excerpts of a session using the multigrid Fourier analysis tool that accompanies the recent book by Wienands and Joppich. Fourier analysis is the main quantitative tool for the development and assessment of multigrid algorithms and for debugging multigrid code. First introduced by Brandt in the 1970s (see [1]) as a predictor of the smoothing properties of relaxation, local mode analysis, as it is commonly called, has been utilized by multigrid developers and practitioners to select subprocesses and parameters in multigrid algorithms and to verify correctness of codes for a very wide range of problems. This includes nonlinear, nonsymmetric, and/or singular-perturbation equations and systems that often prove difficult to analyze, even qualitatively, by other means. Though rigorous only in special cases, the practical utility of local mode analysis is tremendous, and textbooks on multigrid methods often feature one or several chapters on these techniques. On top of that, many papers have been published on various aspects of this subject. And yet, we have before us a complete book devoted solely to this.

Why is such a book warranted? The authors ask this very question in the first page of the preface. Their answer has to do with the software that accompanies the book, **LFA**, and the associated graphical user interface (GUI), **xlfa**. This software provides the user with a first-class quantitative analysis tool for multigrid algorithms. It is reasonably straightforward to apply, especially with the extensive explanations and examples provided in the book, and it includes a substantial number of features. Among these is an impressive and diverse collection of problems already implemented, featuring scalar partial differential equations (PDEs) in two and three dimensions, as well as PDE systems in two dimensions. The features include a wide variety of relaxation methods and intergrid transfers, various coarsening strategies, direct and Galerkin coarse-grid discretizations, and optional parameters for relaxation, residual weighting, and correction weighting. In addition to the problems already implemented, the user can introduce new problems and new coarse-grid correction methods. This is a remarkably useful tool for testing new ideas, verifying old ones, exploring parameter regimes, debugging code, and also for aiding in the teaching of basic and intermediate-level multigrid techniques.

The book itself is comprised of two distinct parts of approximately equal volume. The first part is made up of four chapters. Chapter 1 introduces notation and some basics: iterative methods, Fourier components, and a very brief look at multigrid methods. It also introduces the GUI. Here and later, the book acts in part as a user's guide to the software. Chapter 2 is a very brief description of Fourier analysis for multigrid algorithms, its scope and limitations. Chapter 3 begins with a rather brief introduction to multigrid methods, including a formal definition of the relevant algorithms. It then goes back to the software, in particular the GUI, and describes its features in relation to the different parts of the multigrid algorithm. Thus, at the end of the first three chapters, the reader has a pretty good idea of how to use the software and what it can do. Nevertheless, I found this manner of introduction somewhat unusual. I might have preferred the introduction to multigrid to have been given in a single early chapter (which most potential users could probably skip), followed by a basic chapter on Fourier analysis, and then a user's guide–type chapter that could be employed easily both for learning how to use the software and for back-reference during use. Also, the English, especially in the parts that describe the software, could benefit from some polishing up. Chapter 4, which concludes the first part of the book, is an excellent collection of case studies that demonstrate the features of the software, while also providing a crash course in how to adapt multigrid algorithms to various special difficulties. The problems covered start with simple examples, but quickly go on to anisotropic problems, high-order discretizations, fourth-order equations, singular perturbation problems, three-dimensional applications, and coupled systems of equations.

The second part of the book describes the theory of local mode analysis. While much of the material (excluding the three-grid analysis) can be found in other sources, nowhere is it given in such scope and detail. Chapter 5 is devoted to single-grid (i.e., smoothing) analysis, including the concept of h-ellipticity. Chapter 6 describes two-grid and three-grid analysis in great detail and accuracy. Finally, Chapter 7 discusses rather briefly some more specialized aspects, namely, orders of intergrid transfers, a simplified multigrid Fourier analysis, analysis for cell-centered discretization, and analysis of GMRES preconditioned by multigrid. These subjects are only touched upon, and the interested reader is referred to the literature.

To summarize, I would recommend this book to anyone seriously interested in development of multigrid algorithms for PDEs and systems. The subject of local mode analysis is treated accurately and in great detail, including a large and diverse collection of examples. The software that accompanies the book is of great use and not difficult to master. The book is a fairly easy read for the most part, but it suffers from the lack of a more thorough proofreading. Fortunately, the benefits greatly outweigh this minor deficiency.

REFERENCE

[1] A. Brandt, *Multi-level adaptive solutions to boundary-value problems*, Math. Comp., 31 (1977), pp. 333–390.

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Real Analysis: Measure Theory, Integration, and Hilbert Spaces. *By E. Stein and M. Shakarchi.* Princeton University Press, Princeton, NJ, 2005. $59.95. xx+402 pp., hardcover. ISBN 0-691-11386-6.

The book under review is a text for a first-year graduate course in analysis. However, this text has a rather unique perspective; indeed, as is stated in the foreword, *Real Analysis* is the third volume in the Princeton Lectures in Analysis, a series of four textbooks that aim to present, in an integrated manner, the core areas of analysis. The authors also state that the book serves as a basis for a 48-lecture-hour course.

This book therefore is not designed to cover all topics often found in a first-year graduate course in analysis. Nevertheless, all the topics that are covered are certainly core topics in analysis. The first three chapters deal with Lebesgue measure theory and integration theory, including optional sections on the Brunn–Minkowski inequality, a Fourier inversion formula, the Minkowski content of a curve, and the isoperimetric inequality in the plane.

The next two chapters deal with Hilbert spaces, including sections on Fatou's theorem, the Riesz representation theorem, compact operators, and the Fourier transform on L^2. As a nice application of these methods, they prove a result concerning the existence of solutions to partial differential equations with constant coefficients.

Chapter 5 is concluded with an optional section on the Dirichlet principle, including a treatment of finding a solution by the direct method, which then results in obtaining a weakly harmonic solution. By employing previously proved results, it is concluded that the solution is a classical one. Finally, it is shown that the solution continuously assumes its desired boundary values. This is done only in the plane because the higher-dimensional analogue requires material beyond their scope. However, there is a very accessible treatment of the Wiener criterion for harmonic functions (which is not as well known as it should be) by L. C. Evans and R. Jensen [2]. Also, see G. C. Evans [1].

Chapter 6 serves as a complement to the first three chapters by covering abstract integration theory which includes outer measures, Fubini's theorem, the Radon–Nikodým theorem, as well as an optional section on ergodic theorems.

The last chapter, Chapter 7, deals with fractals and Hausdorff measure. Inevitably, there is a discussion of Hausdorff dimension, self-similarity, and space-filling curves. There is also an interesting treatment of the Radon transform and Besicovitch's Kakeya sets. Their proof of the existence of such sets relies on the concept of self-replicating sets.

As one would expect from these authors, the exposition is, in general, excellent. The explanations are clear and concise with