Exercise 1. Prove that for a subset $A \subset \mathbb{R}^N$, the following statements are equivalent:

1. $A$ is a convex subset of $\mathbb{R}^N$;
2. any convex combination of a finitely number of points in $A$ is in $A$.

Exercise 2. Let $M \subset \mathbb{R}^N$ be a set and $P$ be the set of all affine combinations of elements in $M$. Show that $P$ is an affine subspace of $\mathbb{R}^N$.

Exercise 3. Let $x_1, \cdots, x_n \in \mathbb{R}^N$ be distinct points. Show that there exists $\alpha \in (\mathbb{R}^N)^*$, $\alpha \neq 0$ such that for any $b \in \mathbb{R}$, the hyperplane $H_{\alpha,b}$ contains at most one of the points $x_1, \cdots, x_n$.

Exercise 4. For any subset $M \subset \mathbb{R}^N$, show that:

1. if $0 \in \text{aff}(M)$, then $\dim \text{aff}(M) = \dim \text{span}_\mathbb{R}M$;
2. if $0 \notin \text{aff}(M)$, then $\dim \text{aff}(M) = \dim \text{span}_\mathbb{R}M - 1$. 