Convex polytopes in algebraic combinatorics, WS 17/18
Exercise Sheet 1

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to be handed in on 23, October, 2017 in the lecture.

Exercise 1. Prove that for a subset $A \subset \mathbb{R}^N$, the following statements are equivalent:

1. A is a convex subset of $\mathbb{R}^N$;
2. any convex combination of a finitely number of points in A is in A.

Exercise 2. Let $M \subset \mathbb{R}^N$ be a set and P be the set of all affine combinations of elements in M. Show that P is an affine subspace of $\mathbb{R}^N$.

Exercise 3. Let $\mathbf{x}_1, \cdots, \mathbf{x}_n \in \mathbb{R}^N$ be distinct points. Show that there exists $\alpha \in (\mathbb{R}^N)^*$, $\alpha \neq 0$ such that for any $b \in \mathbb{R}$, the hyperplane $\mathcal{H}_{\alpha,b}$ contains at most one of the points $\mathbf{x}_1, \cdots, \mathbf{x}_n$.

Exercise 4. For any subset $M \subset \mathbb{R}^N$, show that:

1. if $0 \in \mathrm{aff}(M)$, then $\dim \mathrm{aff}(M) = \dim \mathrm{span}_{\mathbb{R}} M$;
2. if $0 \notin \mathrm{aff}(M)$, then $\dim \mathrm{aff}(M) = \dim \mathrm{span}_{\mathbb{R}} M - 1$.