Convex polytopes in algebraic combinatorics, WS 17/18
Exercise Sheet 2

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to be handed in on 6, November, 2017 in the lecture.

Exercise 1. Prove that the crosspolytope

$$C_N^{\Delta} := \mathrm{conv}\{\pm \mathbf{e}_1, \cdots, \pm \mathbf{e}_N\} \subset \mathbb{R}^N$$

admits the following H-description

$$C_N^{\Delta} = \{\mathbf{x} = (x_1, \cdots, x_N) \in \mathbb{R}^N \mid |x_1| + \cdots + |x_N| \leq 1\}.$$

Exercise 2. Let $P \subset \mathbb{R}^N$, $Q \subset \mathbb{R}^M$ be polytopes. Prove that the product $P \times Q \subset \mathbb{R}^{N+M}$ is a polytope.

Exercise 3. Construct an affine surjective map from the Birkhoff polytope $B_N \subset \mathbb{R}^{N^2}$ onto the permutahedron $\Pi_{N-1} \subset \mathbb{R}^N$.

Exercise 4. Consider for $N \geq 1$ the light cone

$$\mathcal{L}_N := \{(\mathbf{x}, t) \in \mathbb{R}^N \times \mathbb{R} \mid \ |\mathbf{x}| \leq t\}.$$

Show that $\mathcal{L}_2$ is not polyhedral.

Exercise 5.

1. Let S be a compact subset of $\mathbb{R}^N$. Prove that the convex hull $\mathrm{conv}(S)$ is a compact subset of $\mathbb{R}^N$.

2. Give a closed subset S in $\mathbb{R}^2$ such that $\mathrm{conv}(S)$ is not closed.