Convex polytopes in algebraic combinatorics, WS 17/18
Exercise Sheet 5

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to be handed in on 18, December, 2017 in the lecture.

Exercise 1. Let P be a simplicial d-polytope.

1. Check that the Dehn-Sommerville equations for $d = 4$ are equivalent to the two linear relations $f_0(P) - f_1(P) + f_2(P) - f_3(P) = 0$ and $f_2(P) = 2f_3(P)$.
2. For $d = 5$, find a linear relation that follows from the Dehn-Sommerville equations but is independent of the Euler formula and $2f_3(P) = 5f_4(P)$.

Exercise 2. Compute the face lattice $\mathcal{L}(C_3^\Delta)$ of the octahedron.

Exercise 3. Let $P \subset \mathbb{R}^M$ and $Q \subset \mathbb{R}^N$ be two polytopes. Show that if P and Q are affinely equivalent, then they are combinatorially equivalent.

Exercise 4. Let $P \subset \mathbb{R}^N$ and $Q \subset \mathbb{R}^M$ be full-dimensional polytopes, both with 0 in the interior.

1. Describe $(P^\circ \times Q^\circ)^\circ$ in the case that P is the interval $[-1, 1]$ and Q is a regular n-gon centered at 0.
2. In the general case, describe the vertices of $(P^\circ \times Q^\circ)^\circ$ in terms of the vertices of the polytopes P and Q.

Exercise 5.

1. Show that if P and Q are d-polytopes, and the face poset of P is a subposet of the face poset of Q, then P and Q are combinatorially equivalent.
2. (optional) Show that a simple d-polytope all of whose 2-faces are quadrangular is combinatorially equivalent to an n-cube.