198

Bsp. 6 Reeller Ansatz: $\alpha_h(t) = \begin{pmatrix} a_1 \cdot e^{\boxed{1\cdot}t} \cos(1 t) + b_1 \cdot e^{1 t} \sin(t) \\ a_2 e^t \cos(t) + b_2 e^t \sin(t) \end{pmatrix}$

$$\Rightarrow \dot{\alpha}_h(t) = \begin{pmatrix} -a_1 \cdot e \sin(t) + b_1 \cdot e \cos(t) + a_1 e^t \cos(t) + b_1 e^t \sin t \\ -a_2 \cdot e \sin(t) + b_2 \cdot e \cos(t) + a_2 e^t \cos t + b_2 e^t \sin t \end{pmatrix}$$

$$\overset{!}{=} A \alpha_h(t) = \begin{pmatrix} a_1 e^t \cos + b_1 e^t \sin - a_2 e^t \cos - b_2 e^t \sin \\ a_1 e^t \cos + b_1 e^t \sin + a_2 e^t \cos + b_2 e^t \sin \end{pmatrix}$$

$$\Rightarrow \begin{cases} \cancel{a_1 e} - a_2 e = b_1 + \cancel{a_1} \\ \cancel{b_1 e} - b_2 e = -a_1 + \cancel{b_1} \\ a_1 + \cancel{a_2} = b_2 + \cancel{a_2} \\ b_1 + \cancel{b_2 e} = -a_2 + \cancel{b_2} \end{cases} \Rightarrow \Rightarrow \begin{matrix} b_1 = -a_2 \\ b_2 = a_1 \end{matrix}$$

$$\Rightarrow \alpha_h(t) = \begin{pmatrix} a_1 e^t \cos(t) - a_2 e^t \sin(t) \\ a_2 e^t \cos(t) + a_1 e^t \sin(t) \end{pmatrix}$$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Inhomog.: Part. Lsg.: $\Phi(t) = \begin{pmatrix} e^t \cos(t) & -e^t \sin(t) \\ e^t \sin(t) & e^t \cos(t) \end{pmatrix}$

$$\Rightarrow \Phi(t)^{-1} = \frac{1}{(e^t)^2} \begin{pmatrix} e^t \cos t & e^t \sin t \\ -e^t \sin t & e^t \cos t \end{pmatrix}$$

$$\rightarrow \alpha_{part.}(t) = \begin{pmatrix} e^t \cos t & -e^t \sin t \\ e^t \sin t & e^t \cos t \end{pmatrix} \int_0^t \frac{1}{e^\tau} \begin{pmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{pmatrix} \begin{pmatrix} \tau - 1 \\ \tau \end{pmatrix} d\tau$$

200

Bsp. 7 $\dot{x} = 2 \frac{x}{\tan(t)} + \sin^3(t) \qquad 0 < t < \frac{\pi}{2}$,

Lineare, inh. DGL mit $a(t) = \frac{2}{\tan t}$, $b(t) = \sin^3(t)$.

$$\alpha_0(t) = \exp\left(\int_{t_0}^t \frac{2}{\tan\tau} d\tau \right) = \exp\left(\int_{t_0}^t \frac{2\cos\tau}{\sin\tau} d\tau \right)$$

$$= \exp\left(\int_{t_0}^t 2 \frac{(\sin\tau)'}{\sin\tau} d\tau \right) = \exp\left(2 \left[\ln(\sin\tau) \right]_{t_0}^t \right)$$

$$= \exp\left(\ln(\sin^2 t) - \ln(\sin^2 t_0) \right) = e^{\ln(\sin^2 t)} / e^{\ln(\sin^2 t_0)}$$

$$= \sin^2 t / \sin^2 t_0$$

$$\alpha(t) = \alpha_0(t) \cdot \Big(x_0 + \underbrace{\int_{t_0}^t \sin^2 t_0 \cdot \sin\tau \, d\tau}_{\sin^2 t_0 \left[-\cos\tau \right]_{t_0}^t} \Big) \qquad x_0 \in \mathbb{R}$$

$$= \frac{\sin^2 t}{\sin^2 t_0} x_0 + \sin^2 t \left(-\cos t + \cos t_0 \right)$$

$$\left[\dot{\alpha} = 2 \frac{\sin t \cos t}{\sin^2 t_0} x_0 + 2 \sin t \cos t \left(-\cos t + \cos t_0 \right) + \sin^2 t \sin t \checkmark \right]$$