Workshop on “Novel Adaptive Discontinuous Galerkin Approaches for the Simulation of Atmospheric Flows”

30. November - 01. December 2022
Room Archimedes, Floor 3, Nr 304
Department of Mathematics and Computer Science
Weyertal 86-90, 50931 Cologne

Wednesday (November 30th)

12:00 - 12:15 Meet and Greet at the Department Mathematik (www.mi.uni-koeln.de/NumSim/how-to-find-us/), Room Archimedes, 3rd Floor, Nr 304

12:15 - 13:30 *Lunch* at Restaurant “Il Cavaliere” (www.ilcavaliere.de)

14:00 – 14:05 *Opening Statement*

14:05 – 15:00 Andrés Rueda-Ramírez (Division of Mathematics, University of Cologne)

A Static Condensation Algorithm for Time-Implicit discretizations of Gauss-Lobatto Discontinuous Galerkin Spectral Element Methods

Abstract - We present a nodal (collocation) time-implicit Discontinuous Galerkin Spectral Element Method (DGSEM) with Gauss-Lobatto (GL) points, and show that it can be formulated as a Schur complement problem and solved using the static condensation method. By doing this, the linear system size is reduced, specially for high orders of accuracy, maintaining the advantageous properties of orthogonal basis expansions. We obtain significant speed-ups as compared to previous explicit and implicit implementations.

15:00 – 16-00 David Knapp (DLR Cologne, Institute for Software Technology)

StartUpDG.jl – A toolbox for rapid DG

Abstract - This talk will be about StartUpDG.jl, a Julia-library providing element-wise operators for a wide range of Discontinuous Galerkin (DG) methods. Specifically, we will present our adaptation of stock differentiation and integration operators of standard elements like tetrahedra and hexahedra for prism-elements. Since StartUpDG.jl also provides logic for constructing and working with unstructured meshes, we have to support meshes consisting of elements with mixed surface types as is the case for prisms. Additionally, we aim to present a prototype solver for DG on prisms based on the tools provided by StartUpDG.jl.

16:00 – 16:30 *Coffee Break*

16:30 – 17:30 Gregor Gassner (Division of Mathematics, University of Cologne)

Robustness, Shock Capturing, Guaranteed Positivity for Discontinuous Galerkin Methods

Abstract - We present recent limiting strategies for DG methods based on compatible subcell finite volume methods that allow for a convex blending of the low order method with the high order DG scheme. With smart choices of the blending coefficients, the resulting hybrid scheme can handle strong shocks and provides guaranteed positivity of the solution, i.e. a non-crashable scheme.

17:30 - 18:00 *Summary of the day / Discussion*

18:30 - 20:00 *Dinner* at Restaurant “Cafe Feynsinn” **(**www.cafe-feynsinn.de**)**

Thursday (December 1st)

09:00 – 10:00 Oswald Knoth (Leibniz Institute for Tropospheric Research)

Test examples from numerical weather prediction in the DG code Fluxo

Abstract - In the talk I give a survey on the implementation of a suite of test examples from numerical weather prediction in the DG Fortran code Fluxo. These examples include two-dimensional slice examples with and without orography and in addition examples on the sphere with a cubed sphere grid. Besides the standard formulation with total energy necessary modifications are described for a change to internal energy and adding of moisture variables. Through numerical experiments we will show that the baroclinic wave test case for a deep atmosphere is only stable for split formulations of the volume kernel. In the second part I describe first attempts to implement split explicit time integration methods and a necessary modified distribution of dg elements for parallel runs.

10:00 – 10:20 Coffee Break

10:20 – 11:20 Michael Baldauf (Deutscher Wetterdienst)

A Discontinuous Galerkin scheme as a possible dynamical core for the forecast model ICON

Abstract - The ICON model is the actual forecast model at the Deutscher Wetterdienst both for global and for regional weather predictions. Its current solver of the Euler equations (a so-called dynamical core), for diffusion, and for tracer advection uses a combination of finite volume/finite difference schemes of second order and is at least mass conserving. At present, an alternative dynamical core based on the Discontinuous Galerkin (DG) method is under development to achieve a higher order discretization both in space and time and to locally conserve not only mass but also momentum and energy. Firstly, the use of local coordinates and related base vectors on a (horizontal) triangle grid on the sphere is described together with a covariant formulation of the equations. The so-called strong conservation form of the equations keeps (almost) local conservation also for the use of terrain-following coordinates. The DG method uses a nodal base on Gauss quadrature points together with IMEX Runge-Kutta schemes. The latter is needed to treat vertically propagating fast waves in an implicit manner by a so-called horizontally explicit-vertically implicit (HEVI) scheme. Results from idealized test cases will be presented that reflect the actual status. Finally, several open questions will be addressed as the overall time integration scheme for the coupling of physical parameterizations on larger time scales, possible stabilization mechanisms, positive-definiteness and mass-consistency of tracer advection and implicitly treated sedimentation.

11:20 – 12:20 Florian Prill (Deutscher Wetterdienst)

Implementing DG-FEM for numerical weather prediction: choices and requirements

Abstract - This presentation describes the parallel numerical toolkit under the hood of our discontinuous Galerkin scheme. While the DWD code "BRIDGE" is still a prototype, its design is yet more formal, as the framework might actually become part of the final implementation in DWD's ICON model. Here, the basic requirements learned from previous throwaway prototypes and the implications of targeting compatibility with the full ICON model are summarized. For applications in numerical weather prediction several aspects are identified as essential, for example a focus on 2.5D discretizations and Kronecker product linear algebra. We discuss these implementation choices and address the dilemma of performance constraints and maintaining flexibility in numerical experiments.

12:40 - 13:45 *Lunch* at Restaurant “Cafe Goldjunge” (cafe-goldjunge.de/cafes/#suelz)

14:00 – 15:00 Aleksey Sikstel (Division of Mathematics, University of Cologne)

A-posteriori error estimator for systems of hyperbolic conservation laws

Abstract - Based on the recent work by Bressan et al. we develop a rigorous a-posteriori error estimator for first-order finite volume schemes approximating solutions of systems of hyperbolic PDEs in one spatial dimension. Our estimator relies on $W^{-1,1}$-norms of the weak PDE residual and the weak entropy dissipation that we estimate by computable expressions. Furthermore, the estimator incorporates oscillations in subdomains of the space-time solution, more specifically in trapezoids containing large isolated jumps and smooth part of the solution, respectively.

15:00 – 15:30 Coffee Break/Discussion

15:30 – 16:30 Johannes Markert (DLR Cologne, Institute for Software Technology)

t8code - the final answer to AMR

Abstract - In this talk, we will present t8code, a C/C++ library we are developing to manage parallel adaptive meshes (AMR) containing various element types. t8code utilizes a collection (a forest) of multiple connected adaptive space-trees in parallel scaling over a million MPI ranks and over 1 trillion mesh elements. Our approach extends the well-known and efficient space-filling curve (SFC) technique for quads and hexes to all commonly used element shapes. Moreover, we will show how t8code's frontend is supposed to be used in scientific simulation codes and we will give details about our implementation from the backend perspective.

16:30 – 17:00 Summary of day 2 / Discussion