

Lattices and Quadratic Forms (Summer 2024) - Extra Problem Set

1. Let C be a binary linear code and $W_C(x, y)$ be its weight enumerator. If $\tilde{C}$ is the corresponding extended code (where a parity bit is appended), find its weight enumerator in terms of that of C .
2. Consider the Hamming code $H := H(\mathbb{F}_2, r)$.
 - a) Show that in the dual code $H^\perp$ every nonzero codeword has weight 2^{r-1} .
 - b) Compute the weight enumerator of H and $H^\perp$ for any r .
3. Find all the possible weight enumerators for self-dual binary linear codes of length 8.
4. Let C be a self-dual, ternary linear code of length 12 with minimum distance > 3 . Find its Hamming weight enumerator. *Hint:* Show that the weights in such code have to be divisible by three.

Remark: The extended ternary Golay code is indeed such a code.