Due in class on June 27, 2024

Lattices and Quadratic Forms (Summer 2024) - Problem Set 8

1. Let C be a binary linear code. Prove that
$$(\Gamma_C)^\sharp = \Gamma_{C^\perp}.$$
2. Let C be a binary $[n, k, d]$ code with .
$$n = 2^r - 1, \quad k = 2^r - 1 - r, \quad \text{and} \quad d = 3.$$
Prove that C is equivalent to the Hamming code $H(\mathbb{F}_2, r)$.
3. Prove that a linear code over $\mathbb{F}_q$ of length n and dimension k with generator matrix $G = [I_k|Q]$ is self-dual if and only if Q is a square matrix with $QQ^T = -I_k$.
4. Let C be a binary linear code that is self-orthogonal. Show that every codeword has even weight. If it is also true that the codewords corresponding to the rows of its generator matrix have weights divisible by four, then show that C is doubly even.

Note: You may use computational tools to help with your solutions, but describe your steps. Each problem is worth 10 points.